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ABSTRACTS

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SPIN ECHOES IN SYSTEMS WITH ANOMALOUS DIFFUSIONS

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In classical Brownian motion of particle the correlations between a past and future movement is zero and

$$\left\langle q^2(t+\Delta t)\right\rangle - \left\langle q^2(t)\right\rangle \equiv \left\langle q^2(\Delta t)\right\rangle , \tag{1}$$

where $q(t)$ is generalized coordinate (angle or vector) which describes the position of the particle in space and $q(\Delta t) = q(t+\Delta t) - q(t)$ is the increment of particle migration. In ordinary (classical) Brownian motion [1,2]

$$\left\langle q^2(t)\right\rangle = 2\alpha \cdot t , \tag{2}$$

where $\alpha = 3D$ for usual diffusion (D denotes the self-diffusion coefficient).

Eq.(1) indicates that in classical Brownian motion

$$\left\langle q(t)q(\Delta t)\right\rangle = 0 , \tag{3}$$

i.e., the increments during ordinary diffusion are uncorrelated and independent of each other.

If there are longer time correlations among the individual movements and Eq.(3) is not fulfiled, then the motion becomes the fractal (anomalous) Brownian process [1,2]. In this case [1-3]

$$\left\langle q^2(t)\right\rangle = 2\alpha \cdot t^{\beta} , \tag{4}$$

where $\beta \le 1$ and α is the constant.

Using Eq.(4) and identity

$$\left\langle q^2(t+\Delta t)\right\rangle \equiv \left\langle q^2(t)\right\rangle + 2\left\langle q(t)q(\Delta t)\right\rangle + \left\langle q^2(\Delta t)\right\rangle ,$$

we have

$$\left\langle q(t)q(\Delta t)\right\rangle \equiv \left\langle q(t)\left[q(t+\Delta t) - q(t)\right]\right\rangle = \alpha\left[(t+\Delta t)^{\beta} - t^{\beta} - (\Delta t)^{\beta}\right] . \tag{5}$$

So as

$$\left\langle q(t')q(t'')\right\rangle \equiv \left\langle q(t')q(t')\right\rangle + \left\langle q(t')\left[q(t'') - q(t')\right]\right\rangle ,$$

using Eqs.(4) and (5) one can easily show that ($t'' > t'$) [3]

$$\left\langle q(t')q(t'')\right\rangle = \alpha\left(t''^{\beta} + t'^{\beta} - \left|t'' - t'\right|^{\beta}\right) . \tag{6}$$

The influence of anomalous diffusion on NMR spin-echo decay in both constant and pulse field-gradient experiments was considered in [3,4]. In these works the expression, which describes the attenuation of echo signal at $t = 2\tau$ was obtained, where τ is delay between RF pulses. However, as it was demonstrated in [5-7] the time position of spin-echo signal in solids with molecular mobility is not coinciding with $t = 2\tau$ and in slow molecular motion region the amplitude of echo signal is observed at $t < 2\tau$. In present communications we considered the time position and decay of the amplitude of spin-echo signal in systems with anomalous diffusion.

Using the procedure, described in [3], we obtained the following expression for the two-pulse echo signal

$$V(t,\tau) = \exp(-\alpha \cdot f(t,\tau,\beta)) \ , \tag{7}$$

where

$$f(t,\tau,\beta) = \frac{1}{\beta+1}\left(4\tau^{\beta+2} + \frac{\beta+3}{\beta+2}t^{\beta+2} - 2\tau \cdot t^{\beta+1} - 2t \cdot \tau^{\beta+1} - \frac{2}{\beta+2}(t-\tau)^{\beta+2}\right) . \tag{8}$$

In [3,4] it was assumed that the maximal amplitude of echo signal is observed at $t = 2\tau$. Inserting $\tau = t/2$ into Eq.(8) gives the result obtained in [3]

$$f(t = 2\tau, \beta) = \frac{3\left(1 - 1/2^{\beta}\right)}{(\beta+1)(\beta+2)} t^{\beta+2} \ .$$

However, from Eqs.(7) i (8) we found out that the maximal amplitude of echo signal is observed when

$$(\beta+3)t^{\beta+1} - 2(\beta+1)\tau \cdot t^{\beta} - 2\tau^{\beta+1} - 2(t-\tau)^{\beta+1} = 0 . \tag{9}$$

From Eq.(9) it follows, that when $\beta = 1$ (classical diffusion), for the time position of amplitude of echo signal t_e, we have: $t_e = \sqrt{2}\tau \neq 2\tau$.

Fig.1 and Fig.2 illustrate the dependences of time position and amplitude of echo signal on τ .

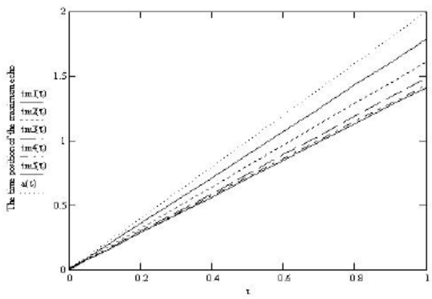

Fig.1. The dependence of the time position of maximum echo-signals on τ . $tm1(\tau)$ - $\beta = 0.1$; $tm2(\tau)$ - $\beta = 0.25$; $tm3(\tau)$ - $\beta = 0.5$; $tm4(\tau)$ - $\beta = 0.75$; $tm5(\tau)$ - $\beta = 1$; $a(\tau) - t = 2\tau$

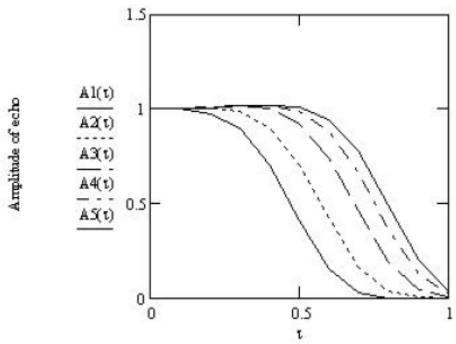

Fig.2. The dependence of echo amplitude on τ . $A1(\tau)$ - $\beta = 0.1$; $A2(\tau)$ - $\beta = 0.25$; $A3(\tau)$ - $\beta = 0.5$; $A4(\tau)$ - $\beta = 0.75$; $A5(\tau)$ - $\beta = 1$

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