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*ABSTRACTS*

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## KUBO - ANDERSON OSCILLATOR AND NMR DYNAMICAL PROBLEMS

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The Kubo-Anderson oscillator is described by equation [1,2]

$$\dot{x} = i\alpha(t) \cdot x, \quad (1)$$

where  $\alpha(t)$  is a stochastic function of the time and the distribution of all possible values  $\alpha(t)$  is described by the function  $p(\alpha)$ . The jumps from one value  $\alpha(t)$  to other are independent and distributed uniformly over the time with density  $\nu_0$  (the value  $\nu_0 dt$  determines the average jumps value happen on the time interval  $dt$ ).

The average solution of Eq.(1) has the form

$$\langle x(t) \rangle = \left\langle x(0) \cdot \exp\left(i \int_0^t \alpha(t') dt'\right) \right\rangle, \quad (2)$$

where symbol  $\langle \dots \rangle$  means an operation of averaging over all realizations of the random process.

In NMR (and EPR) there are many dynamical problems, which may be described by the Eq.(1). We consider only some of these problems.

1. The shape of free induction decay  $G(t)$  (or the shape of NMR line  $I(\omega)$ ) for the case when NMR resonance frequency  $\omega(t) \equiv \alpha(t)$  is the random function of the time [3,4]. This random time dependence of  $\omega(t)$  is appeared as a result of spectral or usual (spatial) diffusion. In this case  $x(t) = G(t)$ , where  $G(t)$  is the shape of free induction decay (FID).

2. The shape of spin echo signals in spin systems with stochastic NMR resonance frequency  $\omega(t)$  [3,4]. Here also as in the case of FID random time dependence of  $\omega(t)$  is appeared as a result of spectral (spin) or usual (spatial) diffusion. For example, for the case of two-pulse echo the echo signal is determined by equation [3,4]

$$\langle V_E(\tau, t) \rangle = \left\langle \exp\left(i \int_0^\tau \omega(t') dt' - i \int_\tau^t \omega(t') dt'\right) \right\rangle. \quad (3)$$

3. The dipolar correlation function  $h(t)$  in the case of non-Markovian molecular mobility [5]

$$h(t) = \overline{M_2} - (M_2 - \overline{M_2}) \cdot \left\langle \exp\left(- \int_0^t \nu_C(t') dt'\right) \right\rangle. \quad (4)$$

Here  $\overline{M_2}$  is the second moment of motionally narrowed NMR line;  $M_2$  is the second moment of NMR spectrum of "rigid" lattice;  $\nu_C(t)$  is the random depended correlation frequency which describes the molecular mobility. For this case in Eq.(1) we have  $\alpha(t) \equiv i\nu_C(t)$ .

The general solution of Eq.(1) can be easily obtained using the method of the differentiation formulae described in [6]. Using this method we obtain

$$\frac{d}{dt}\langle x_k \rangle = \langle x_{k+1} \rangle - \nu_0 \langle x_k \rangle + \nu_0 \cdot \langle x \rangle \cdot \langle (i\alpha)^k \rangle. \quad (5)$$

Here

$$x_k = (i\alpha)^k \cdot x. \quad (6)$$

The Laplace transformation of Eq.(5) has the form

$$z \cdot \bar{x}_k - \langle x_k(0) \rangle = \bar{x}_{k+1} - \nu_0 \bar{x}_k + \nu_0 \cdot \overline{\langle (i\alpha)^k \rangle \cdot \langle x \rangle}, \quad (7)$$

where

$$\overline{\langle (i\alpha)^k \rangle \cdot \langle x \rangle} = \int_0^\infty e^{-z t} \langle (i\alpha)^k \rangle \cdot \langle x \rangle \cdot dt, \quad \bar{x}_k = \int_0^\infty e^{-z t} \cdot \langle x_k \rangle dt. \quad (8)$$

Assuming that the stochastic process is stationary process ( $\langle (z + \nu_0 + i\alpha(t))^{-1} \rangle \equiv \langle (z + \nu_0 + i\alpha(0))^{-1} \rangle$ ) and using Eq.(7) we obtain

$$\bar{x}_0 = \frac{\int_0^\infty \frac{p(\alpha) \cdot x(0)}{z + \nu_0 + i\alpha(0)} \cdot d\alpha}{1 - \nu_0 \cdot p(z)}, \quad p(z) = \int_0^\infty \frac{p(\alpha) d\alpha}{z + i\alpha(0) + \nu_0} \quad (9)$$

In Eq.(9) the function  $p(\alpha)$  describes the distribution of  $\alpha(0)$ .

For the case when frequency jumps between two frequencies  $\alpha(t) = \omega(t) = \pm \Delta$   $\{p(\omega) = [\delta(\omega + \Delta) + \delta(\omega - \Delta)]/2\}$  and  $x(0) = 1$  from Eq.(11) it follows the well known result for NMR line shape [7].

Using Eq.(9) for Laplace transformation of two-pulse echo signal we obtain

$$\begin{aligned} V_E(s, z) &= \int_0^\infty d\tau \cdot e^{-s\tau} \int_0^\infty dt \cdot e^{-z t} \cdot V_E(\tau, t) = \\ &= \int_0^\infty \frac{p(\omega) \cdot d\omega}{(z + \nu_0 - i\omega) \cdot (s + \nu_0 + i\omega)} \cdot \frac{1}{[1 - \nu_0 \cdot p(s)] \cdot [1 - \nu_0 \cdot p(z)]}. \end{aligned} \quad (10)$$

For the case, when  $p(\omega) = [\delta(\omega + \Delta) + \delta(\omega - \Delta)]/2$  from Eq.(10) it follows the well known result for the shape of two-pulse echo signal [8].

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