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ABSTRACTS

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DECAYS OF CARR-PURCELL ECHOES

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In this report, we consider the decays of spin echoes after the $90^0_Y - [\tau - 180^0_X - \tau - \text{echo}]_n$ pulse sequence applied to spin systems with strong inhomogeneous broadening of the resonance lines. The case of $n = 1$ corresponds to the well-known classical Hahn echo [1]. For $n > 1$ this pulse sequence produces the echo signals at $t = 2\tau, 4\tau, \ldots, 2n$ as it was shown by Carr and Purcell [2, 3]. In our consideration, it has been assumed that the resonance (NMR or EPR) frequency ω is a stochastic function of time. The calculation of the echo decay was reduced to the calculation of the average value [4]

$$\upsilon(t) = \left\langle \exp\left[i\int_0^t s(t')\omega(t')dt' \right] \right\rangle , \tag{1}$$

where angular brackets $\langle \cdots \rangle$ denote the average over stochastic process and "echo function" $s(t')$ depends on the n and has the values ± 1 [4].

Using the method of calculation described in [4] and assuming that the stochastic process may be considered as Gauss-Markovian process [4], we obtained the following results:

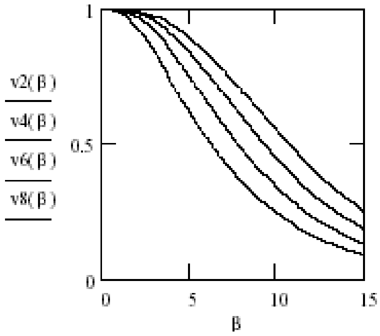
$$\upsilon(t = 2\tau) = \exp[-\sigma^2\tau_C^2(-3 + \frac{t}{\tau_C} + 4\exp\left(-\frac{t}{2\tau_C}\right) - \exp\left(-\frac{t}{\tau_C}\right))] , \tag{2}$$

$$\begin{aligned} \upsilon(t = 4\tau) = \exp[-\sigma^2\tau_C^2(-5 + \frac{t}{\tau_C} + 4\exp\left(-\frac{t}{4\tau_C}\right) \\ + 4\exp\left(-\frac{t}{2\tau_C}\right) - 4\exp\left(-\frac{3t}{4\tau_C}\right) + \exp\left(-\frac{t}{\tau_C}\right))] \end{aligned} , \tag{3}$$

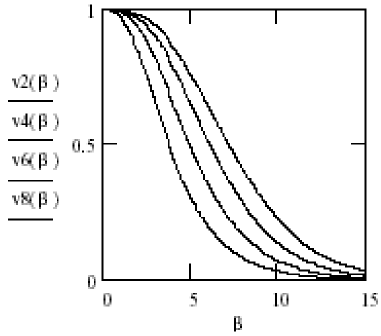
$$\begin{aligned} \upsilon(t = 6\tau) = \exp[-\sigma^2\tau_C^2(-7 + \frac{t}{\tau_C} + 4\exp\left(-\frac{t}{5\tau_C}\right) + 8\exp\left(-\frac{t}{3\tau_C}\right) \\ - 4\exp\left(-\frac{t}{2\tau_C}\right) - 4\exp\left(-\frac{2t}{3\tau_C}\right) + 4\exp\left(-\frac{5t}{6\tau_C}\right) - \exp\left(-\frac{t}{\tau_C}\right))] \end{aligned} , \tag{4}$$

$$\begin{aligned} \upsilon(t = 8\tau) = \exp[-\sigma^2\tau_C^2(-9 + \frac{t}{\tau_C} + 4\exp\left(-\frac{t}{8\tau_C}\right) + 12\exp\left(-\frac{t}{4\tau_C}\right) - 4\exp\left(-\frac{3t}{8\tau_C}\right) \\ - 8\exp\left(-\frac{t}{2\tau_C}\right) + 4\exp\left(-\frac{5t}{8\tau_C}\right) + 4\exp\left(-\frac{3t}{4\tau_C}\right) - 4\exp\left(-\frac{7t}{8\tau_C}\right) + \exp\left(-\frac{t}{\tau_C}\right))] \end{aligned} . \tag{5}$$

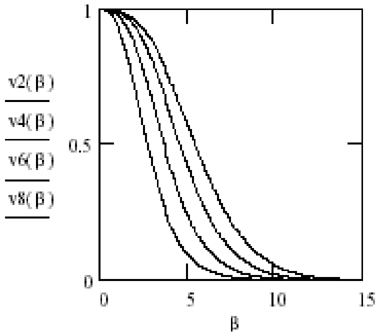
Here τ_c is the correlation time, which describes the stochastic process, and σ determines the inhomogeneous broadening of the resonance line. Eq. (2) represents the well known result [4], while Eqs. (3) – (5) represent new results. The dependences of echo signals ($v2(\beta) \equiv \upsilon(t = 2\tau)$, $v4(\beta) \equiv \upsilon(t = 4\tau), \ldots$) on $\beta = t / \tau_C$ are plotted in figures.



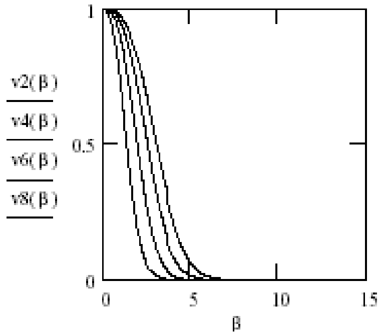
$\sigma^2 \tau_C^2 = 0.2$



$\sigma^2 \tau_C^2 = 0.5$



$\sigma^2 \tau_C^2 = 1$



$\sigma^2 \tau_C^2 = 5$

From Eqs. (2) – (5) we obtain that in the case $\tau > \tau_c$

$$\upsilon(t = 2n\tau) = \exp\{-(\sigma^2 \tau_c) \cdot t\} \equiv \exp(-t / T_2) \ , \tag{6}$$

so in this case the decays of all Carr-Purcell echoes are described by the same spin-spin relaxation time $T_2^{-1} = \sigma^2 \tau_c$. This result was not noted in [1-4].

In the case $\tau < \tau_c$ from Eqs.(2) – (5) we have

$$\upsilon(t = 2n\tau) = \exp\left\{-\frac{1}{(2n)^2}\frac{\sigma^2}{3\tau_c} \cdot t^3\right\} \ . \tag{7}$$

At the certain definite value of time ($t = \text{const}$) from Eq.(7) it follows that $\upsilon(t = 2n\tau) > \upsilon[t = (2n-1)\tau)]$, and so the echo signal at $t = 2n\tau$ damps slowly than the echo signal observed at $t = (2n - 1)\tau$. This is also the well known result obtained for the case of diffusion in inhomogeneous magnetic field [1-3].

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